Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Journal of Sound and Vibration 293 (2006) 901–909

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

A test rig to investigate slab track structures for controlling ground vibration

S.J. Cox[a,*], A. Wang[a], C. Morison[a], P. Carels[b], R. Kelly[b], O.G. Bewes[c]

[a]*Pandrol Rail Fastenings Ltd, 63 Station Road, Addlestone, Surrey KT15 2AR, UK*
[b]*CDM Inc., Reutenbeek 9-11, B3090 Overijse, Belgium*
[c]*ISVR, University of Southampton, Southampton SO17 1BJ, UK*

Accepted 26 August 2005

Abstract

This paper describes a test rig developed within the CONVURT research project in an attempt to bridge the gap between bench top tests on individual components and full-scale installation on service track. A large test rig has been constructed based on three 2.5 m long full-scale floating track slab elements. These can be directly connected to the base slab, or resiliently supported. Two 12 m long rails are attached to the slab elements with different fastener systems that can provide a wide range of stiffness values. The track can be pre-loaded, and the protruding ends of the rail are damped to reduce end reflections. The primary purpose of the test rig is to measure the dynamic properties of a range of different floating slab and direct fixation fastening systems. A mathematical track model has been used to extract dynamic parameters of the resilient elements in the systems, which are then used to predict performance in track.

---

1. Introduction

There is increasing interest around the world in ground-borne vibration from railway systems, and how this can be controlled through the track structure. The focus of this paper is on vibrations emanating from underground railways. The characteristics of vibrations from railways that cause disturbance in buildings have been identified [1]. The frequency range of greatest concern is usually between about 40 and 120 Hz. Standards exist that allow criteria for acceptable vibration levels to be determined [2]. There is an understanding of how track structure relates to performance in controlling vibration. Ref. [3] identifies that “The broad principles of vibration isolation of railways consist of the reduction of the dynamic stiffness of the track support, and further, the introduction or increase in the mass of elements of the tracks support, plus adjustments to damping”.

Non-ballasted track (slab track) is often favoured in railway tunnels, because of its potential to reduce maintenance costs. Slab track requires a degree of resilience to control vibration. A wide range of solutions are available, providing different levels of performance. Direct fixation fastening (DFF) systems, which are

---

*Corresponding author. Tel.: +44 1932 834500; fax: +44 1932 850858.
E-mail address: s.cox@pandrol.com (S.J. Cox).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2005.08.051

usually fitted on to a rigid slab base are relatively inexpensive, and can be retrofitted to existing tracks if necessary. Floating slab tracks (FST) with significant added mass provide superior performance but are expensive and cannot normally be retrofitted after initial construction.

An accurate knowledge of the comparative level of performance of different track structures is necessary to select cost effective solutions. It is sometimes possible to replace one DFF system with another in the track, and to measure the effect on vibration levels directly. Measurements of this type [4] have suggested a 13 dB reduction in vibration levels on the tunnel invert for each ten-fold decrease in DFF system dynamic stiffness. But directly comparable measured data on the performance of fundamentally different track forms is rare, and comparisons between implementations in different circumstances, where different measurement methods have been employed, are potentially misleading.

Measurements made at any one location cannot in any case be used to accurately extrapolate to find vibration levels in different circumstances—for example where different train speeds, soil conditions, or distances between source and receiver apply. Mathematical models allow theoretical comparison and evaluation of different track support systems. Such models require as input an accurate description of the track structure. Methods exist by which some of the important parameters required, such as the dynamic stiffness and damping, can be measured for individual components in the laboratory. For example direct and indirect methods are proposed in the European CEN standards [5]. However, whole track support systems, such as FST and ballasted track, cannot easily be tested in the laboratory. The scope of the CEN standard states that it applies to measurement of DFF systems, but not systems that incorporate concrete elements or other high mass elements.

Parameters that represent the dynamic behaviour of existing whole track systems can be extracted from measurements made on the track by tuning of mathematical models of the track system [6]. However, tests and changes in track components on operating railways can be very difficult to arrange. Because of the high cost of track and the necessity to meet safety and reliability criteria, experimentation with new track forms—or even significant variation from existing proven track forms—is relatively rare. A gap exists between bench top tests on individual components and full-scale installations on service tracks.

Large laboratory test rigs incorporating lengths of floating slab have been constructed in the past. For example, a 14.8 m long section of floating slab was used to investigate the performance of different designs of FST bearing elements [7]. No investigations were made into the relative performance of floating slab and direct fixation fastener track forms, or into the most effective forms of direct fixation fastener for use on top of floating slab track.

2. The CONVURT test rig

This paper presents a large-scale test rig that has been developed with the aim of providing an intermediate step between laboratory scale tests and track installations. CONVURT [8] was a research project, partly funded by the European Union, into a range of issues related to ground-borne vibration from trains running in tunnels. The programme included modelling, measurements, and the development of solutions and of standards. The test rig was a result of collaboration between two companies that participated in CONVURT and that are represented by the authors. Pandrol is a supplier of DFF systems. CDM is a supplier of the elastic materials used in DFF systems, and a designer and supplier of FST systems.

The primary purpose of the test rig is to measure the dynamic properties of a range of different track systems. Changes in track configuration and measurements can be made much more quickly and easily than could be done in track, and all of the measurements are made on the same basis. Simple methods can be used to extract parameters that are similar to those used with measurements on existing tracks [6]. The rig can also be used to measure the effect of changes in track configuration on the distribution of applied loads, and on the deflection of component parts of the track in response to loading. This information can be fed back into the design of improved track systems. The test rig does not aim to reproduce a section of slab track in a tunnel, but is a relatively simple system into which real track components can be introduced and evaluated. Nevertheless, it is desirable that the dynamic characteristics of the rig should be broadly similar to those of a typical slab track. Comparisons between some measured characteristics of the rig and the corresponding characteristics measured in a real tunnel are made below.

2.1. Description of test rig

The test rig, shown in Fig. 1, has been constructed from three 2.4 m long full-scale floating track slab elements. These are to a proprietary CDM design. Each element has a width of 2.5 m, a thickness of 0.3 m, and a mass of approximately 5000 kg. The elements can be connected with L-section steel plates that run along the top outer edges of the slabs and bridge the joints. The connectors, which are bolted to the top and sides of the elements, are 0.9 m long, 0.15 m wide and 12 mm thick. In all of the tests described here, the elements were joined. The slab elements can be raised up and pads or springs installed beneath so that they are resiliently supported, or lowered so that they are directly resting on the base slab. This last configuration has been used to extract parameters for DFF track systems. In practice, slab tracks without floating elements would have a monolithic slab directly connected into the tunnel floor. The floating slab elements in the rig have lateral supports that can be fitted with resilient side bearings. However in all of the tests described here, these lateral bearings were omitted.

Two 12 m long rails are attached to the slab elements at standard track gauge with different fastener systems that can provide a wide range of stiffness values. The rails can, in principle, be changed although in all the tests described here, S49 rails were used. The rails overhang at each end of the test slab. Sand boxes were installed at

Fig. 1. A photograph of the CONVURT test rig.

Table 1
The different track form configurations tested

Configuration	Description
FST1	CDM Elastomer strips—higher stiffness
FST2	CDM Elastomer strips—medium stiffness
FST3	CDM Elastomer strips—low stiffness
FST4	CDM Elastomer pads—low stiffness
FST5	CDM Elastomer mats—low stiffness
DFF1	Simple Pandrol SFC fastener with stiff railpad
DFF2	Simple Pandrol SFC fastener with soft railpad
DFF3	Low stiffness Pandrol VIPA SP two layer resilient baseplate
DFF4	Very low stiffness Pandrol VANGUARD resilient rail chair

each end of each rail to damp out reflections from the ends of the rails. This is discussed further below. There are four fasteners on each floating slab unit spaced at 0.6 m apart, so that there are 12 fasteners in total supporting each rail. The fasteners used on the test rig are Pandrol designs.

The test rig has a loading frame above its centre that allows a static load to be applied with a hydraulic cylinder through a spreader beam to each of the rails. The frame and cylinder have been designed for loadings of up to 300 kN. Provision has been made for an isolation spring between the spreader beam and the cylinder to isolate the loading arrangement from the rest of the test rig. In its current configuration, the CDM spring nest, which is primarily designed for use in base isolation of buildings, is too stiff to adequately isolate the frame. In future its stiffness will be reduced so that dynamic tests can be carried out in a loaded condition, but in this paper only unloaded dynamic test results are reported.

A brief description of the nine track form configurations tested using the rig is shown in Table 1. The actual stiffness values that were extracted from the rig for each configuration are discussed below. Test configurations in which floating slab track elements were resiliently mounted have been denoted as FST*n* where *n* identifies the configuration; and those with no resilience are DFF*n* (after Direct Fixation Fastener). The fastening system used to attach the rail to the slab for all of the FST configurations was the DFF1 system.

2.2. Characteristics of the test rig

2.2.1. Static loading

For each test configuration the loading frame was used to apply a force to the rails. Deflections of the top slab relative to the base, and the rail relative to the top slab were measured at a number of positions. Measurements for FST track types for the top slab relative to the base are shown in Fig. 2(a) below, and for DFF track types for the rail relative to the top slab in Fig. 2(b). Deflections of the top slab for DFF configurations were negligible and are not shown. Rail deflections for FST track types were very similar to the results shown for DFF1, and are not shown. Note that there is no fastening at the centre of the rig directly beneath the loading position, and it can be seen from Fig. 2(a) that the FST slab bends under loading, so that for some support configurations the peak deflection occurs under the fastener positions, where the load is transmitted through the fastener to the slab, rather than at the centre of the rig beneath the rail loading. The results show that there is no change of gradient at the ends of the FST sections, which indicates an effective transfer of shear loads and bending moments between the elements.

2.2.2. Dynamic characteristics

The primary purpose of the rig is to determine system parameters from measured dynamic characteristics, such as the natural frequencies at which the rail decouples from the slab for all configurations and at which the slab decouples from the rig base for the FST cases, to allow determination of the dynamic stiffness of the

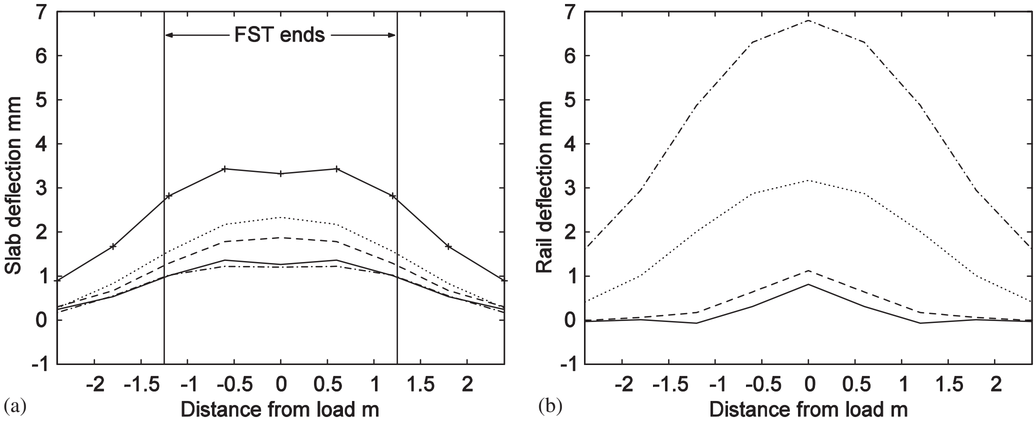

Fig. 2. Deflected shapes under static loading: (a) deflection of slab relative to the invert: —, FST1; – –, FST2; · · ·, FST3; –·–, FST4; –+–, FST5. (b) Deflection of the rail relative to the slab: —, DFF1; – –, DFF2; · · ·, DFF3; –·–, DFF4.

resilient elements in the system [6]. Because there is no equivalent to the unsprung mass of the train on the test rig, these modes will occur at higher frequencies than those that would be associated with the same components in track under traffic. In the figures below, the dynamic characteristics of the rig are shown for a frequency range that extends to 400 Hz. This is wider than the range of greatest interest for vibration control measures fitted to slab track, which usually need to be most effective in the frequency range between about 40 and 120 Hz.

Fig. 3 shows the narrow-band direct vertical rail head receptance (displacement of rail per unit input force as a function of frequency) measured on the rig for the DFF4 configuration. The measurement was obtained by applying a dynamic force at the centre of one rail using a large soft-tipped calibrated hammer and measuring the resulting vibration response of the rail with accelerometers.

Pandrol and CDM worked with another CONVURT project partner, the Milan metro system ATM, to make two installations of the Pandrol VANGUARD DFF fastener in track [9]. The track form for the installation made on Line 2 in Milan is the same as the DFF4 configuration of the test rig. The receptance of the track measured in Milan is also shown in Fig. 3. The rail head receptances measured in track and on the rig are generally similar, although the rig characteristic includes a number of peaks at higher frequencies. These are associated with reflections from the ends of the rails. The DFF4 track configuration is that which has the lowest decay rate (i.e. where vibrations are transmitted furthest along the rail) among those tested and therefore where end reflections are most apparent. The sand box dampers that were installed on the rig

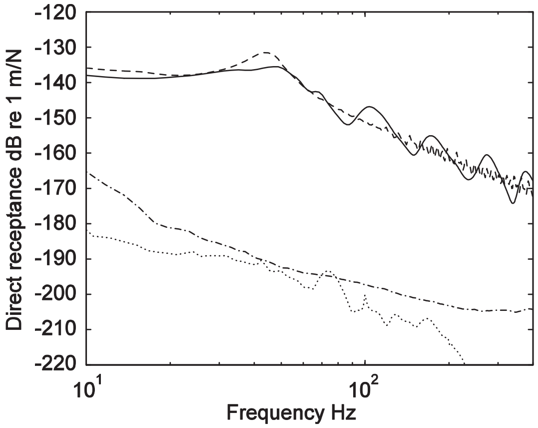

Fig. 3. Direct receptances measured on the test rig with the DFF4 track form: —, rail head receptance on test rig; – –, rail head receptance measured on track; · · ·, test rig base receptance; –·–, tunnel floor receptance measured on track.

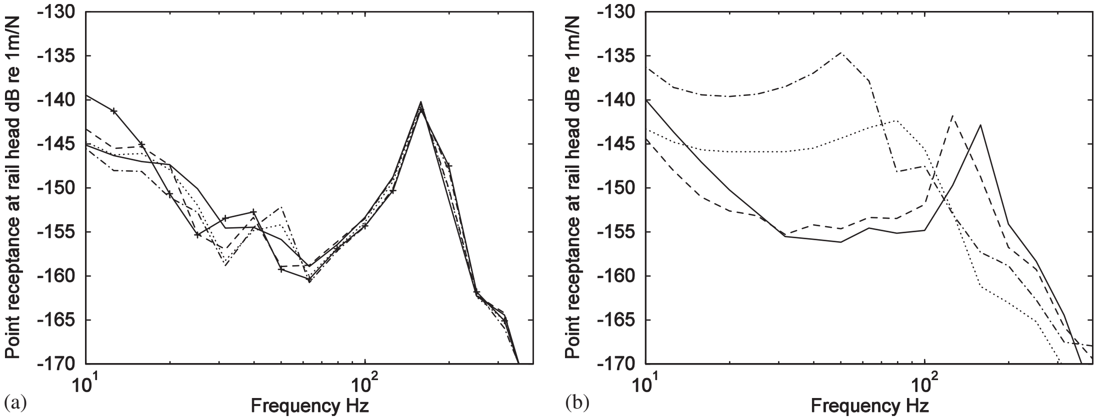

Fig. 4. Point receptance measured at the rail head: (a) floating slab configurations: —, FST1; – –, FST2; · · ·, FST3; –·–, FST4; –+–, FST5. (b) Direct fastening configurations: —, DFF1; – –, DFF2; · · ·, DFF3; –·–, DFF4.

reduced the amplitude of the end reflections to a level where aspects of the characteristic that can be used to extract system parameters—principally resonances of the masses or the rail and slab on the stiffness of the resilient elements in the system—were clearly distinguishable in the response.

Fig. 3 also shows the direct receptance of the base of the test rig, and of the invert of a typical metro system bored tunnel as measured on site on Guangzhou metro. The receptance of the base of the rig is broadly similar to that of a typical tunnel. Furthermore, the receptance of the rig is sufficiently low that a large impedance mismatch exists at the connection between the FST and the rig base meaning that the dynamic parameters extracted from the tests can be considered as independent of the base impedance.

The rail receptance measurements were repeated for all of the track configurations described in Table 1. This data is presented in Fig. 4. For clarity, results for FST track types are shown in Fig. 4(a) and those for DFF track types are shown in Fig. 4(b). Three features of particular interest can be identified:

- All of the FST types and one of the DFF types show a distinct peak in the response at 160 Hz. These are all of the track forms that share the common rail fastener type DFF1. Investigation confirmed that this peak corresponds to a mode where the rail vibrates on the stiffness of the fastener. For other DFF track types which have a lower fastener stiffness, the corresponding mode occurs at lower frequencies, as is shown in Fig. 4(b).
- For the four FST track types there is a peak in each transfer function at frequencies of between 12.5 and 25 Hz that corresponds to the vibration mode where the floating slab vibrates as a rigid body on the stiffness of its resilient supports. The frequency of the peak varies according to the stiffness of the supports. These modes are not present for DFF track types.
- For the four FST track type only, there is a peak in the response at 40–50 Hz. Measurements suggest that this is a bending mode of the slab. The mode is not present when the slab is not resiliently supported. The frequency of this mode is not strongly influenced by the stiffness of the slab resilient supports.

3. Use of the test rig to extract parameters

In principle, a number of different methods can be applied to extract parameters to represent the resilient elements of the test rig. Here a model developed within the CONVURT project by the French consultancy Vibratec [10] has been used to extract parameters. The model is an 'excitation' model of the vehicle–track system and can be used to predict the rail head receptance of the track. The model is described in more detail below. As the geometrical and material properties of the rail and slab were known, the values of the dynamic stiffness and damping of each resilient element were adjusted to obtain a best fit between model prediction and the measured data presented in Fig. 4. A limitation of this method is that it is only possible to obtain a single value for the dynamic stiffness at a single frequency. However, the dynamic stiffness is not expected to vary significantly with frequency in the range presented.

The values for the dynamic stiffness and damping loss factor of the track elements found using this method for all of the different configurations tested are shown in Table 2. Note that the parameters shown represent the track in an unloaded condition. Stiffness values are given per metre of rail length, so that the relative stiffness of fasteners and FST resilient elements can be compared.

The dynamic stiffness for the DFF4 track form given above equates to 3.2 kN/mm per fastener. The stiffness of this same DFF4 system had previously been estimated from unloaded track receptance measurements made on track on Line 2 in Milan [9]. The estimated stiffness of the fastener was 2.7 kN/mm. The stiffness of an individual fastener similar to DFF4 (though designed for a larger rail section and known to have a slightly higher static stiffness) has been measured in the laboratory by the driving point method [11] and the stiffness was found to be 5.3 kN/mm [12].

The damping factors extracted for the resilient slab supports in the FST configurations were consistently high, and higher than the known internal damping of the FST bearing elements when these are tested in isolation. Other sources of damping—such as friction—must contribute to the overall damping factor for the FST track structure, which is the parameter that is extracted by the method described here.

Table 2
The values for the dynamic stiffness of the resilient elements per rail calculated from receptance data for each track form configuration

Configuration	FST stiffness (kN/mm/m)	FST damping loss factor	DFF stiffness (kN/mm/m)	DFF damping loss factor
FST1	12.5	0.5	—	—
FST2	8.0	0.5	—	—
FST3	6.4	0.5	—	—
FST4	10.0	0.5	—	—
FST5	5.0	0.5	—	—
DFF1	—	—	50	0.07
DFF2	—	—	30	0.1
DFF3	—	—	15	0.3
DFF4	—	—	5.3	0.3

4. Calculations made based on parameters drawn from the test rig

The parameters extracted from the rig shown in Table 2 have then been used to predict vibration levels generated by railway traffic in existing tunnels. An example is given here, in which the 'excitation' model of the vehicle–track system [10] developed within the CONVURT project by Vibratec has been used to predict vibration levels on a tunnel floor. In principle, models that include the 'propagation' of vibration beyond the track can be used to predict vibration levels in buildings. Models of this type were also developed within the CONVURT project [13,14].

Several models were developed by Vibratec within the CONVURT project. That used here is a model for slab track in which the rail and slab are modelled as two infinite beams. The rail fasteners and the support of the lower slab are modelled as layers of continuous damped stiffness. The rolling stock is modelled as a series of masses, springs, and dampers. The track is assumed to be straight. Coupling between wave propagation in the soil and in the track is not accounted for. Excitation is by roughness on the wheels and rails. The dynamic force at the wheel/rail contact point is calculated from the rolling stock receptance, the track receptance, and the combined wheel and rail roughness spectrum. The track response and force acting on the track bed at any position is then calculated.

The Vibratec model has been used to estimate vibration levels on the floor of a tunnel for each of the track configurations tested. The dynamic stiffness and damping parameters of the resilient track and slab supports were as found from the test rig and shown in Table 2. Parameters used to represent the rolling stock were obtained from ATM. The rail roughness input spectrum was obtained from measurements made on the London Underground [15]. The measured roughness spectrum contained wavelengths of up to 10 cm. To extend the excitation frequency range an assumed roughness level was extrapolated from the spectrum for wavelengths up to 0.7 m. Using a train speed of 11 m/s resulted in an excitation input down to approximately 16 Hz. The forces output from the model were used to calculate the vibration on the tunnel floor using the measurement of the receptance of a typical tunnel structure shown in Fig. 3. The calculated vibration spectra represent those that would be expected to occur on the base of a tunnel during train pass-by for different track forms. The vibration spectra produced are shown in Fig. 5(a) for FST type systems and in Fig. 5(b) for DFF type systems.

In the case of the FST track forms as shown in Fig. 5(a), the most prominent feature is a peak in the vibration level at 63 Hz, which is also shared (though less clearly) with the spectrum for the DFF1 system. This is associated with a mode in which the vehicle unsprung mass vibrates on the stiffness of the fastener. In all these cases, since the fastener stiffness is the same, this mode occurs at the same frequency. This peak detracts from the performance of the FST system in the most important frequency range. To investigate this further, additional calculations have been made for FST track configurations in combination with different track fastener stiffness levels. Two additional spectra are plotted in Fig. 5(a) showing configuration FST1 in combination with fastener stiffness DFF4 (i.e. a much lower value) and also in combination with a fastener with a much higher stiffness—five times higher than the value extracted here for DFF1. The lower fastener stiffness gives the best result, particularly at the higher frequencies where lower vibration levels then are obtained with even the softest DFF track form are seen.

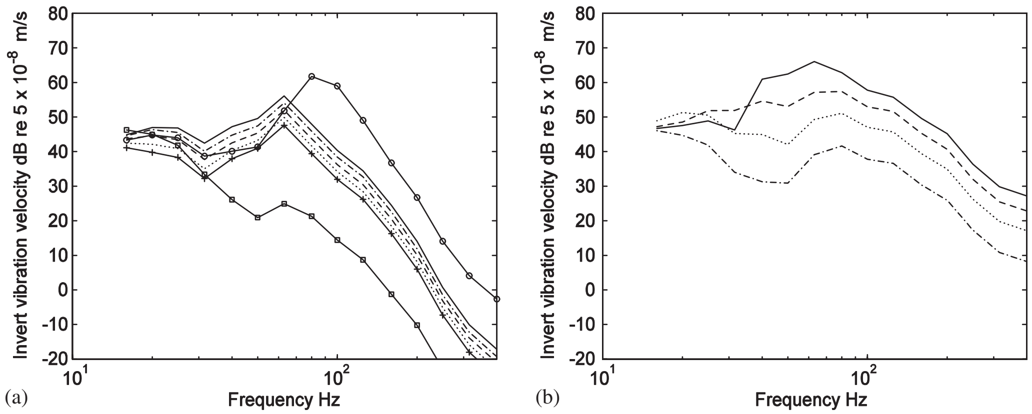

Fig. 5. Predicted vibration levels on tunnel floor: (a) floating slab configurations: —, FST1; – –, FST2; · · ·, FST3; –·–, FST4; –+–, FST5; –○–, FST1 and hard pad; –□–, FST1 and DFF4. (b) Direct fastening configurations: —, DFF1; – –, DFF2; · · ·, DFF3; –·–, DFF4.

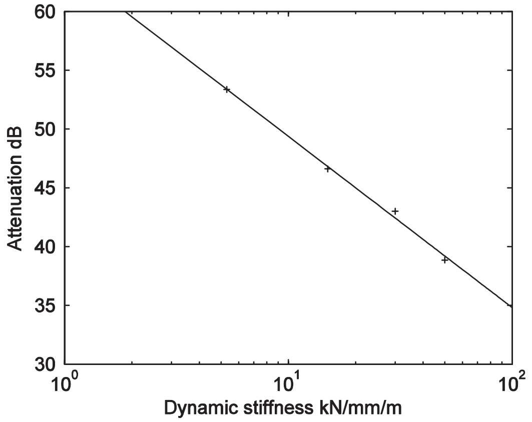

Fig. 6. Predicted effect of fastener stiffness on total vibration attenuation between rail and tunnel floor: +, predicted with Vibratec model; —, best fit.

In the case of the DFF track forms, there is a clear improvement in performance as the stiffness of the fastener is reduced, with lower vibration levels predicted across the whole spectrum, but particularly in the frequency range between 40 and 120 Hz—the range in which waves that propagate from the tunnel to the surface are most commonly found. To investigate this further, the total vibration attenuation between the rail and tunnel floor (calculated from the difference in the total vibration velocity levels on the rail and the tunnel floor) for each DFF track form has been calculated across the whole frequency range presented in Fig. 5. These results are plotted in Fig. 6 below against the dynamic stiffness of the track configuration. The total attenuation level decreases by approximately 14 dB for each ten-fold increase in fastener stiffness. Although individual attenuation levels do not agree with measurements for many different cases, the predicted decrease in vibration levels for a given decrease in track stiffness on the test rig is similar to that previously estimated from a number of measurements on track [4], where the attenuation level was found to decrease by approximately 13 dB for each ten-fold increase in track support stiffness.

5. Conclusions

A test rig developed within the CONVURT project has been described. The rig can be configured with different floating slab track support elements and with different rail fastener systems that can provide a wide range of stiffness values. Dynamic stiffness and damping parameters for the resilient elements in nine track form configurations have been extracted from the test rig. The vehicle–track interaction model developed by Vibratec within the CONVURT project has been used to find the parameters by tuning the rail head receptance predicted by the model to give the best fit with measured results. The timescales and expenses involved in the tests were a fraction of that which would have been required to obtain the same amount of data from a working FST system on track.

The same model has then been used to predict base slab vibration spectra during train pass-by for different track configurations. The track parameters used in these predictions were those extracted from the rig. Vehicle data and rail roughness were taken from measurements made on track, and the predicted tunnel floor forces were applied to a tunnel floor mobility measurement made in Guangzhou to estimate tunnel floor pass-by vibration levels. Results of the predictions suggest that floating slab track forms perform better in important parts of the frequency range (40–100 Hz) when fitted with soft rail fasteners (less than 20 kN/mm stiffness) than with stiffer rail fasteners (greater than 50 kN/mm stiffness). The model of the vehicle–track interaction system predicts a similar rate of change of attenuation between the rail and tunnel floor with track stiffness to that estimated from measurements on track.

Further investigations are planned using the rig with a pre-load applied to the different track systems in order to better evaluate and compare different track structures under loaded track conditions.

References

[1] Federal Transport Administration, Transit Noise and Vibration Impact Assessment, US Department of Transportation, April 1995.

[2] ISO 2631-2:1989, Evaluation of human exposure to whole-body vibration, Continuous and shock-induced vibration in buildings (1–80 Hz), 1989.

[3] ANC Guidelines, Measurement and Assessment of Groundborne Noise and Vibration, ISBN 0-9539516-1-8.

[4] S.J. Cox, A. Wang, Effect of track stiffness on vibration levels in railway tunnels, *Journal of Sound and Vibration* 267 (2003) 565–573.

[5] Railway applications—Track—Performance requirements for fastening systems—Part 6: Special fastening systems for attenuation of vibration—Annex A—Measurement of transfer stiffness". Draft ENV 13481-6:2002(E).

[6] S.L. Grassie, Dynamic modelling of railway track and wheelsets, in: *Proceedings of the Second International Conference on Recent Advances in Structural Dynamics*, 1984.

[7] A. Castellani, G. Kajon, P. Panzeri, P. Pezzoli, Elastomeric materials used for vibration isolation of railway lines, *Journal of Engineering Mechanics* 1 (1998) 614–621.

[8] CONVURT project reference website: www.convurt.com.

[9] S.J. Cox, Installation and testing of vibration reducing track fastenings within the CONVURT project. *Tenth International Congress on Sound and Vibration*, Stockholm, Sweden, July 2003.

[10] CONVURT PROJECT: Development of the excitation model (WP3), VIBRATEC Report 450.003.RA.03.B, December 2003.

[11] Acoustics and vibration—Laboratory measurement of vibro-acoustic transfer properties of resilient elements—Part 1: Principles and guidelines. Draft ISO/FDIS 10846-1:1997(E).

[12] C. Morison, A. Wang, O. Bewes, Methods for measuring the dynamic stiffness of resilient rail fastenings for low frequency vibration isolation of railways their problems and possible solutions, *Journal of Low Frequency Noise, Vibration and Active Control* 24 (2005) 107–116.

[13] D. Clouteau, R. Othman, M. Arnst, H. Chebli, G. Degrande, R. Klein, P. Chatterjee, B. Jansssens, A numerical model for ground-borne vibrations from underground railway traffic based on a periodic FE-BE formulation, *Journal of Sound and Vibration* (2006), this issue, doi:10.1016/j.jsv.2005.12.023.

[14] J.P. Talbot, H.E.M. Hunt, M.F.M. Hussein, A prediction tool for the optimisation of maintenance activity to reduce disturbance due to ground-borne vibration from underground railways, in: *Proceedings of the Eighth International Workshop on Railway Noise*, Buxton, UK, September 2004.

[15] G. Degrande, P. Chatterjee, R. Klein, W. Van de Velde, P. Holscher, V. Hopman, A. Wang, N. Dadkhah, Vibrations due to a test train at variable speeds in a deep bored tunnel embedded in London clay, *Journal of Sound and Vibration* (2006), this issue, doi:10.1016/j.jsv.2005.08.039.